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Effect of variable thermophysical properties on laminar free convection of gas

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Abstract—The thermal conductivity and dynamic viscosity are assumed to vary with absolute temperature according to a simple power law. The density is taken as inversely proportional to absolute temperature at constant pressure, while the Prandtl number is assumed constant. The governing equations for laminar free convection of gas are changed to dimensionless ordinary equations by similarity transformation. They are solved by a shooting method. The numerical calculation results are given along with a detailed discussion.

1. INTRODUCTION

A WELL-KNOWN analysis of the variable fluid property problem for laminar free convection on an isothermal vertical flat plate has been presented by Sparrow and Gregg [1] with solutions of the boundary layer equations for special cases. Brown [2] studied the effect of the coefficient of volumetric expansion on laminar free convection heat transfer. Gray and Giogini [3] discussed the validity of the Boussinesq approximation and proposed a method for analysing natural convection flows with fluid properties assumed to be linear functions of temperature and pressure. Clausing and Kempka [4] reported their experimental study of the influence of property variations on natural convection and concluded that, for the laminar region, Nu_f will be a function of $Ra_f(=Gr_f Pr_f)$ only with reference temperature, T_f, taken as the average temperature in the boundary layer. The instability of laminar free convection flow and transition to a turbulent state had been studied by Gebhart [5] and summarized in a textbook by Eckert and Drake [6].

The present work treats the viscosity and thermal conductivity as $\mu \approx T^{n_\mu}$ and $\lambda \approx T^{n_\lambda}$, respectively. The density is taken as inversely proportional to the absolute temperature, while the specific heat at constant pressure, c_p, and the Prandtl number, Pr, are both assumed constant for analysing laminar free convection of gas.

2. BASIC CONSIDERATIONS OF THE VARIABLE PROPERTIES

The thermodynamic temperature of the gas far away from the wall, T_∞, can be taken as the reference temperature, T_0, for free convection analysis. So we assume

$$\mu/\mu_\infty = (T/T_\infty)^{n_\mu} \quad (1)$$

$$\lambda/\lambda_\infty = (T/T_\infty)^{n_\lambda} \quad (2)$$

while the change of density with thermodynamic temperature at constant pressure can be expressed as

$$\rho/\rho_\infty = (T/T_\infty)^{-1} \quad (3)$$

or

$$\nu/\nu_\infty = (T/T_\infty)^{n_\mu+1}. \quad (4)$$

According to the summarized experimental values of μ and λ for several monoatomic and diatomic gases, and also for air and water vapour, reported in Hisenrath *et al.* [7], n_μ and n_λ values are given in Table 1. The percentage deviations for predicted values of μ/μ_∞ and λ/λ_∞ from equations (1) and (2) are shown in Figs. 1–5.

The Prandtl number is defined as $Pr = \mu c_p/\lambda$. Strictly speaking, Pr should also depend on temperature. However, it is well known that $Pr \approx 0.72$ for diatomic gases, and $Pr \approx 0.7$ for air. Hence, Pr can be taken as a constant for a given gas in the temperature range from T_w to T_∞. Therefore, if we assume

$$c_p/c_{p\infty} = (T/T_\infty)^{n_{c_p}} \quad (5)$$

then

$$n_{c_p} = n_\lambda - n_\mu. \quad (6)$$

It can be found from Table 1 that the values of n_{c_p} are much lower than 0.1 for monoatomic gases, and around 0.1–0.16 for diatomic gases, air and water

NOMENCLATURE

a	diffusivity [$m^2\ s^{-1}$]
c_p	specific heat at constant pressure [$J\ kg^{-1}\ K^{-1}$]
g	gravitational acceleration [$m\ s^{-2}$]
$Gr_{x,\infty}$	local Grashof number, $gx^3(T_w/T_\infty - 1)/v_\infty^2$
n_λ	temperature exponents for thermal conductivity of gas
n_μ	temperature exponent for dynamic viscosity of gas
Nu_x	local Nusselt number, $\alpha_x x/\lambda$
p	pressure [$N\ m^{-2}$]
Pr	Prandtl number, $\mu c_p/\lambda$
q_x	local heat transfer rate per unit area from wall to fluid [$W\ m^{-2}$]
T	absolute temperature [K]
w_x, w_y	velocity component in the x- and y-directions, respectively [$m\ s^{-1}$]
W_x, W_y	dimensionless velocity component in the x- and y-directions, respectively.

Greek symbols

α_x	local heat transfer coefficient [$W\ m^{-2}\ K^{-1}$]
δ	boundary layer thickness [m]
θ	dimensionless temperature, $(T-T_\infty)/(T_w-T_\infty)$
λ	thermal conductivity [$W\ m^{-1}\ K^{-1}$]
μ	absolute viscosity [$kg\ m^{-1}\ s^{-1}$]
v	kinematic viscosity, μ/ρ [$m^2\ s^{-1}$]
ρ	density [$kg\ m^{-3}$].

Subscripts

w	at wall
∞	far from the wall surface.

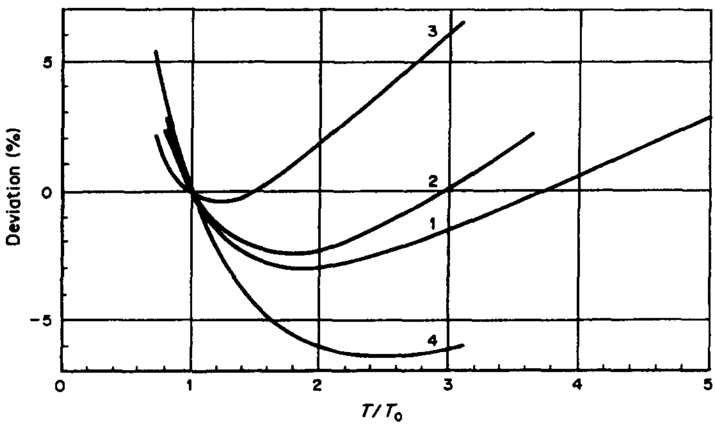

FIG. 1. Deviation of predicted values of μ and λ for air: (1) for evaluation of μ with n_μ; (2) for evaluation of λ with n_λ; (3) for evaluation of μ with n_μ; (4) for evaluation of λ with n_λ.

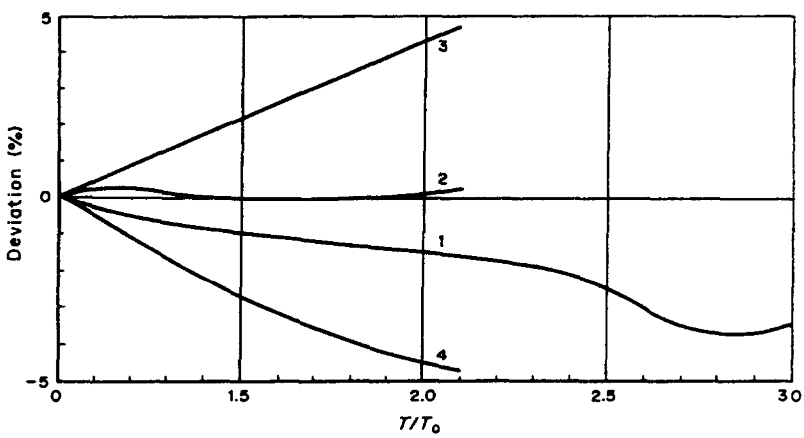

FIG. 2. Deviation of predicted values of μ and λ for water vapour: (1) for μ evaluated with n_μ; (2) for λ evaluated with n_λ; (3) for μ evaluated with $n_{\mu\lambda}$; (4) for λ evaluated with $n_{\mu\lambda}$.

Table 1. The value of parameters n_μ, n_λ and $n_{\mu\lambda}$ from experimental data of μ and λ with deviation

Gas	T_0 (K)	Data source	n_μ	Temperature range (K)	Deviation (%)	n_λ	Temperature range (K)	Deviation (%)	$n_{\mu\lambda}$	Temperature range (K)	Deviation (%)	
											for μ	for λ
Air	273	(7)	0.68	220–1400	±3	0.81	220–1000	±2.5	0.75	200–850	±6.5	±6.5
Water vapour	380		1.04	380–1500	±4	1.185	380–800	±0.3	1.12	380–800	±5	±5
CO	273		0.71	230–1500	±2	0.83	220–600	±2	0.78	210–600	±3	±4
N_2	273		0.67	220–1500	±3	0.76	220–1200	±3	0.72	200–1000	±5.5	±7
O_2	273		0.694	230–2000	±2	0.86	220–600	±2	0.79	200–600	±5	±6
H_2	273		0.68	80–1000	±2	0.8	220–700	±1	0.75	180–700	±6	±6
Ar	273		0.72	220–1500	±3	0.73	210–1500	±4	0.726	210–1500	±3.5	±4
He	273	(8)	0.66	273–873	±1	0.725	273–873	±0.3	0.7	273–873	±4	±4
Ne	273		0.649	273–873	±0.4	0.71	273–873	±0.5	0.68	273–873	±4	±3.5

vapour. For the case $1/2 \leqslant (T/T_\infty) \leqslant 2$, it is possible to treat c_p as a constant value, so as to simplify the analysis but still suit the needs for engineering application.

With Pr and c_p both assumed constant, μ/λ = constant, and therefore it is logical, for diatomic gases, air and water vapour, to take some mean value of n_μ and n_λ as $n_{\mu\lambda}$, such that $n_\mu \approx n_{\mu\lambda} \approx n_\lambda$. We try to express the parameter $n_{\mu\lambda}$ by a weight sum of n_μ and n_λ as

$$n_{\mu\lambda} = 0.45 n_\mu + 0.55 n_\lambda. \tag{7}$$

The deviations for evaluation of μ and λ with $n_{\mu\lambda}$ for diatomic gases, air and water vapour are listed in Table 1 and also plotted in Figs. 1–4.

3. GOVERNING EQUATIONS

The analytical model and coordinating system used for free convection of gas on an isothermal vertical flat plate is shown in Fig. 6. The boundary layer is laminar when $Ra(=Gr\,Pr)$ is less than 10^9 [6].

The conservation equations for mass, momentum and energy of steady laminar flow in the boundary layer for vertical gas free convection are as follows:

$$\frac{\partial}{\partial x}(\rho w_x) + \frac{\partial}{\partial y}(\rho w_y) = 0 \tag{8}$$

$$\rho\left(w_x \frac{\partial w_x}{\partial x} + w_y \frac{\partial w_x}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu \frac{\partial w_x}{\partial y}\right) + \rho g \frac{T - T_\infty}{T_\infty} \tag{9}$$

$$\rho c_p \left(w_x \frac{\partial T}{\partial x} + w_y \frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right). \tag{10}$$

The boundary conditions are

$$y = 0, \quad w_x = 0, \quad w_y = 0, \quad T = T_w \tag{11}$$

$$y \to \infty, \quad w_x \to 0, \quad T \to T_\infty. \tag{12}$$

The partial differential equations (8)–(10) can be transformed to the corresponding dimensionless ordinary differential equations by the following similarity transformations

$$\eta = \frac{y}{x}\frac{(Gr_{x,\infty})^{1/4}}{\sqrt{2}} \tag{13}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}. \tag{14}$$

The velocity components w_x and w_y can be expressed in terms of the dimensionless variables as

$$W_x = [2\sqrt{(gx)}(T_w/T_\infty - 1)^{1/2}]^{-1} w_x \tag{15}$$

$$W_y = [2\sqrt{(gx)}(T_w/T_\infty - 1)^{1/2}(\tfrac{1}{4}Gr_{x,\infty})^{-1/4}]^{-1} w_y. \tag{16}$$

Then, equations (8)–(10) are transformed to dimensionless ordinary differential equations as follows:

$$\left(2W_x - \eta \frac{dW_x}{d\eta} + 4\frac{dW_y}{d\eta}\right) - \frac{1}{\rho}\frac{d\rho}{d\eta}(\eta W_x - 4W_y) = 0 \tag{17}$$

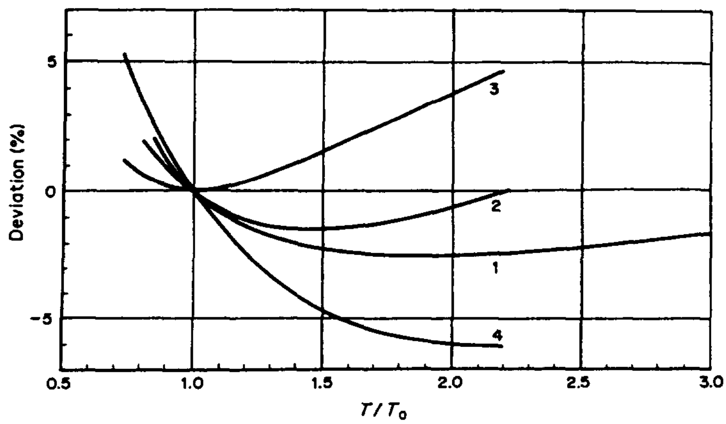

FIG. 3. Deviation of predicted values of μ and λ for O_2: (1) for μ evaluated with n_μ; (2) for λ evaluated with n_λ; (3) for μ evaluated with $n_{\mu\lambda}$; (4) for λ evaluated with $n_{\mu\lambda}$

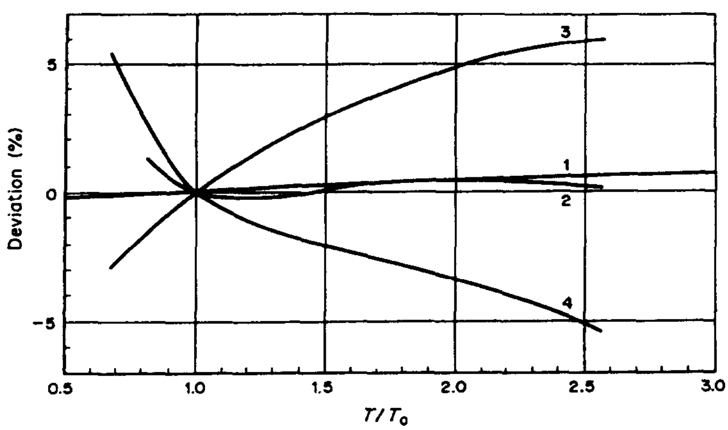

FIG. 4. Deviation of predicted values of μ and λ for H_2: (1) for evaluation of μ with n_μ; (2) for evaluation of λ with n_λ; (3) for evaluation of μ with $n_{\mu\lambda}$; (4) for evaluation of λ with $n_{\mu\lambda}$.

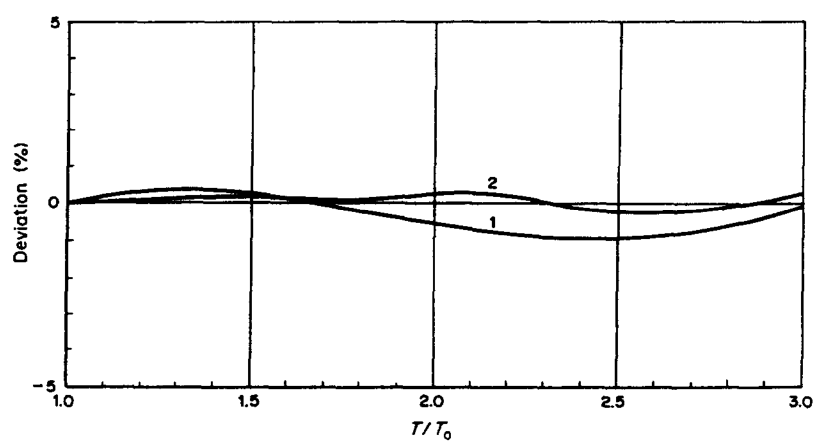

FIG. 5. Deviation of predicted values of μ and λ for He: (1) for μ evaluated with n_μ; (2) for λ evaluated with n_λ.

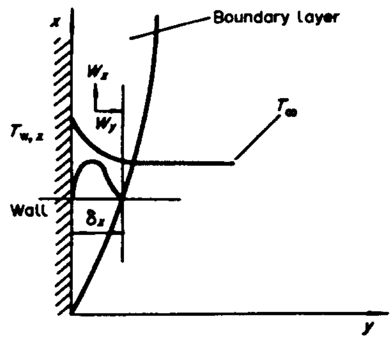

FIG. 6. Physical model coordinate system.

$$\frac{\nu_\infty}{\nu}\left[W_x\left(2W_x-\eta\frac{\mathrm{d}W_x}{\mathrm{d}\eta}\right)+4W_y\frac{\mathrm{d}W_x}{\mathrm{d}\eta}\right] = \frac{\mathrm{d}^2W_x}{\mathrm{d}\eta^2}+\frac{1}{\mu}\frac{\mathrm{d}\mu}{\mathrm{d}\eta}\frac{\mathrm{d}W_x}{\mathrm{d}\eta}+\frac{\nu_\infty}{\nu}\theta \quad (18)$$

$$Pr\frac{\nu_\infty}{\nu}(-\eta W_x+4W_y)\frac{\mathrm{d}\theta}{\mathrm{d}\eta}=\frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2}+\frac{1}{\lambda}\frac{\mathrm{d}\lambda}{\mathrm{d}\eta}\frac{\mathrm{d}\theta}{\mathrm{d}\eta} \quad (19)$$

with boundary conditions

$$\eta=0,\quad W_x=0,\quad W_y=0,\quad \theta=1 \quad (20)$$

$$\eta\to 0,\quad W_x\to 0,\quad \theta\to 0. \quad (21)$$

From equations (1)–(4) combined with equation (14), we have

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}\eta}=-\frac{(T_w/T_\infty-1)\,\mathrm{d}\theta/\mathrm{d}\eta}{(T_w/T_\infty-1)\theta+1} \quad (22)$$

$$\frac{1}{\mu}\frac{\mathrm{d}\mu}{\mathrm{d}\eta}=\frac{n_\mu(T_w/T_\infty-1)\,\mathrm{d}\theta/\mathrm{d}\eta}{(T_w/T_\infty-1)\theta+1} \quad (23)$$

$$\frac{1}{\lambda}\frac{\mathrm{d}\lambda}{\mathrm{d}\eta}=\frac{n_\lambda(T_w/T_\infty-1)\,\mathrm{d}\theta/\mathrm{d}\eta}{(T_w/T_\infty-1)\theta+1} \quad (24)$$

$$\frac{\nu_\infty}{\nu}=[(T_w/T_\infty-1)\theta+1]^{-(n_\mu+1)}. \quad (25)$$

As a modification for diatomic gases, air and water vapour, n_μ and n_λ can be replaced by $n_{\mu\lambda}$ from equation (7).

4. HEAT TRANSFER ANALYSIS

The local heat transfer rate per unit area from the surface of the plate to the gas can be calculated by Fourier's law as

$$q_x=-\lambda_w\left.\frac{\partial T}{\partial y}\right|_{y=0}$$

or

$$q_x=-\lambda_w(T_w-T_\infty)(\tfrac{1}{4}Gr_{x,\infty})^{1/4}x^{-1}\left.\frac{\mathrm{d}\theta}{\mathrm{d}\eta}\right|_{\eta=0}. \quad (26)$$

The local Nusselt number, defined as

$$Nu_{x,\infty}=\frac{\alpha_x x}{\lambda_\infty}=\frac{q_x x}{\lambda_\infty(T_w-T_\infty)}$$

will be

$$Nu_{x,\infty}=-\frac{\lambda_w}{\lambda_\infty}(\tfrac{1}{4}Gr_{x,\infty})^{1/4}\left.\frac{\mathrm{d}\theta}{\mathrm{d}\eta}\right|_{\eta=0}. \quad (27)$$

From equation (2), we have

$$Nu_{x,\infty}=-(T_w/T_\infty)^{n_\lambda}(\tfrac{1}{4}Gr_{x,\infty})^{1/4}\left.\frac{\mathrm{d}\theta}{\mathrm{d}\eta}\right|_{\eta=0}. \quad (28)$$

It is obvious that the velocity and temperature fields can be solved from the governing ordinary differential equations (17)–(19) with boundary conditions, equations (20) and (21), combined with equations (22)–(25), and so we can obtain the $Nu_{x,\infty}$ value from equation (28). It is expected that, for the case of constant properties, the dimensionless velocity field W_x and the dimensionless temperature field θ will be functions of Pr only, but for the case of variable properties, both the dimensionless velocity field and the dimensionless temperature field will depend not only on Pr but also on n_μ, n_λ and (T_w/T_∞).

The calculations were carried out numerically by the shooting method of ref. [9]. The typical results for the velocity and temperature field are plotted in Figs. 7–10. It was found from the numerical calculations that, even for the diatomic gases, air and water vapour, the modification with n_μ and n_λ replaced by $n_{\mu\lambda}$ are unnecessary, because the numerical results obtained either with the actual n_μ and n_λ or with the modified $n_{\mu\lambda}$ are almost the same.

Using the curve matching method, we have

$$-\left.\frac{\mathrm{d}\theta}{\mathrm{d}\eta}\right|_{\eta=0}=\psi(Pr)\left(\frac{T_w}{T_\infty}\right)^{-m} \quad (29)$$

$$\psi(Pr)=0.567+0.186\ln Pr \quad (30)$$

$$m=0.64n_{\mu\lambda}+0.36=0.35n_\lambda+0.29n_\mu+0.36 \quad \text{for } T_w/T_\infty>1 \quad (31)$$

$$m=0.76n_{\mu\lambda}+0.24=0.42n_\lambda+0.34n_\mu+0.24 \quad \text{for } T_w/T_\infty<1. \quad (32)$$

Hence, equation (28) can be rewritten as

$$Nu_{x,\infty}=\frac{\psi(Pr)}{\sqrt{2}}(Gr_{x,\infty})^{1/4}\left(\frac{T_w}{T_\infty}\right)^{n_\lambda-m}. \quad (33)$$

The predicted results of equation (29) are compared with those of the numerical solution from equations (17)–(25), as shown in Table 2. The agreement is quite good.

5. DISCUSSION

For the special case $T_w/T_\infty\to 1$, equation (33) can be simplified to

$$Nu_{x,\infty}=\frac{\psi(Pr)}{\sqrt{2}}(Gr_{x,\infty})^{1/4}. \quad (34)$$

This corresponds to the classical solution with Boussinesq's approximation in ref. [10].

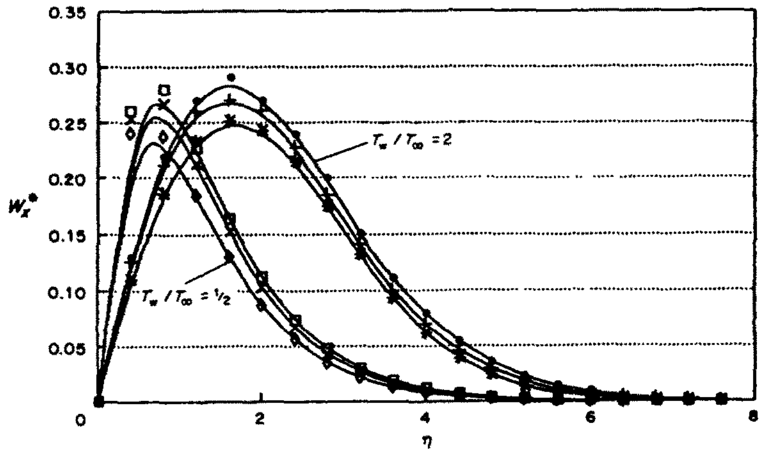

FIG. 7. Comparison of velocity profiles for free convection of different gases: ●—●, □—□, Ar ($Pr = 0.622$, $n_\lambda \approx n_\mu \approx n_{\mu\lambda}$); +—+, ×—×, O_2 ($Pr = 0.733$, $n_{\mu\prime} = 0.79$); *—*, ◇—◇, water vapour ($Pr = 1$, $n_{\mu\prime} = 1.12$).

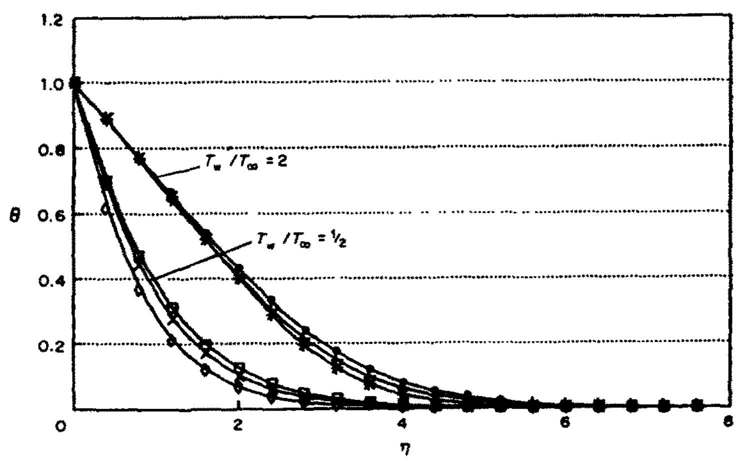

FIG. 8. Comparison of temperature profiles for free convection of different gases: ●—●, □—□, Ar ($Pr = 0.622$, $n_\lambda \approx n_\mu \approx n_{\mu\lambda}$); +—+, ×—×, O_2 ($Pr = 0.733$, $n_{\mu\prime} = 0.79$); *—*, ◇—◇, water vapour ($Pr = 1$, $n_{\mu\prime} = 1.12$).

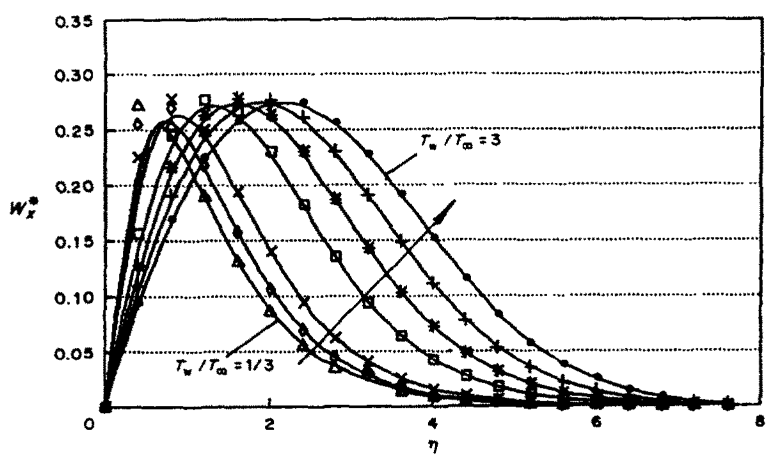

FIG. 9. Comparison of velocity profiles for free convection of air ($Pr = 0.7$, $n_{\mu\prime} = 0.79$) with different T_w/T_∞.

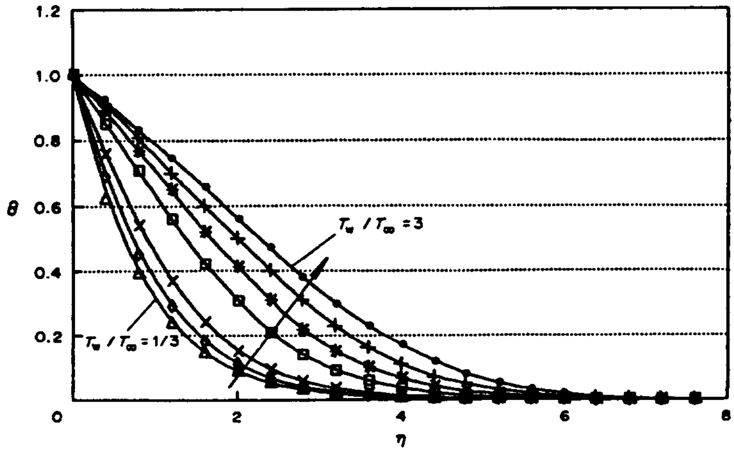

FIG. 10. Comparison of temperature profiles for free convection of air ($Pr = 0.7$ and $n_{\mu\lambda} = 0.79$) with different T_w/T_∞.

For gases with $n_\mu = n_\lambda = 1$, $m \to 1$ from both equations (31) and (32), and so the exponent of T_w/T_∞ in equation (33), $(n_\lambda - m)$, comes to zero. Then, $Nu_{x,\infty}/(Gr_{x,\infty})^{1/4}$ will be the same for different (T_w/T_∞) values, and equation (34) holds true.

Introducing

$$Nu_{x,w} = \alpha_x x/\lambda_w$$

and

$$Gr_{x,w} = gx^3(T_w/T_\infty - 1)/\nu_w^2$$

we have

$$Nu_{x,w} = Nu_{x,\infty}(\lambda_\infty/\lambda_w)$$

$$(Gr_{x,w})^{1/4} = (Gr_{x,\infty})^{1/4}(\nu_\infty/\nu_w)^{1/2}$$

and so

$$\frac{Nu_{x,w}}{(Gr_{x,w})^{1/4}} = \frac{Nu_{x,\infty}}{(Gr_{x,\infty})^{1/4}}\left(\frac{T_w}{T_\infty}\right)^{(n_\mu+1)/2-n_\lambda}$$

or, from equation (33)

$$Nu_{x,w} = \frac{\psi(Pr)}{\sqrt{2}}(Gr_{x,w})^{1/4}\left(\frac{T_w}{T_\infty}\right)^{(n_\mu+1)/2-m}. \quad (35)$$

For the cases of either $T_w/T_\infty \to 1$ or $n_\mu = n_\lambda = 1$, equation (35) is simplified to the form

$$Nu_{x,w} = \frac{\psi(Pr)}{\sqrt{2}}(Gr_{x,w})^{1/4}. \quad (36)$$

The calculated results from equation (36) for the supposed gases A, B and C agree very well with

Table 2. Calculated results of $-(d\theta/d\eta)|_{\eta=0}$: (1) the numerical solution of governing equations; (2) from equation (29) with (31); and (3) from equation (30) with (32)

T_w/T_∞		Ar $Pr = 0.622$ $n_\mu = 0.72$ $n_\lambda = 0.73$	H_2 $Pr = 0.68$ $n_\mu = 0.68$ $n_\lambda = 0.8$	Air $Pr = 0.7$ $n_\mu = 0.68$ $n_\lambda = 0.81$	N_2 $Pr = 0.71$ $n_\mu = 0.67$ $n_\lambda = 0.76$	CO $Pr = 0.72$ $n_\mu = 0.71$ $n_\lambda = 0.83$	O_2 $Pr = 0.733$ $n_\mu = 0.694$ $n_\lambda = 0.86$	Water vapour $Pr = 1$ $n_\mu = 1.04$ $n_\lambda = 1.185$
3	(1)	0.1940	0.1974	0.1987	0.2043	0.1973	0.1973	0.1738
	(2)	0.1935	0.1975	0.1988	0.2044	0.1975	0.1975	0.1738
5/2	(1)	0.2256	0.2300	0.2316	0.2374	0.2306	0.2307	0.2110
	(2)	0.2249	0.2300	0.2318	0.2374	0.2308	0.2311	0.2115
2	(1)	0.2714	0.2772	0.2794	0.2852	0.2792	0.2796	0.2679
	(2)	0.2703	0.2772	0.2796	0.2850	0.2794	0.2801	0.2689
3/2	(1)	0.3438	0.3526	0.3557	0.3609	0.3570	0.3582	0.3651
	(2)	0.3427	0.3527	0.3561	0.3609	0.3575	0.3590	0.3665
5/4	(1)	0.3990	0.4105	0.4144	0.4188	0.4172	0.4193	0.4448
	(2)	0.3983	0.4109	0.4151	0.4191	0.4179	0.4201	0.4459
3/4	(1)	0.6035	0.6276	0.6351	0.6336	0.6446	0.6507	0.7775
	(3)	0.6011	0.6247	0.6333	0.6312	0.6423	0.6479	0.7761
1/2	(1)	0.8344	0.8774	0.8898	0.8776	0.9093	0.9225	1.2181
	(3)	0.8285	0.8666	0.8786	0.8684	0.8993	0.9098	1.2081
1/3	(1)	1.1492	1.2247	1.2448	1.2124	1.2812	1.3075	1.9188
	(3)	1.1419	1.2022	1.2209	1.1949	1.2591	1.2774	1.8805

Table 3. The comparison between the calculated results from equation (35) and these quoted from ref. [1]

Gas	T_w/T	$Nu_{x,w}/(Gr_{x,w})^{1/4}$ From equation (35)	$Nu_{x,w}/(Gr_{x,w})^{1/4}$ Ref. [1]
Gas A [1]	3	0.368	0.368
$Pr = 0.7$	5/2	0.366	0.366
$n_\lambda = n_\mu = 3/4$	2	0.363	0.363
	3/2	0.359	—
	3/4	0.348	0.348
	1/2	0.340	0.339
	1/3	0.332	0.330
Gas B [1]	3	0.373	0.373
$Pr = 0.7$	5/2	0.369	—
$n_\lambda = n_\mu = 2/3$	2	0.366	—
	3/2	0.361	—
	3/4	0.346	—
	1/2	0.336	—
	1/3	0.325	—
Gas C [1]	3	0.354	0.353
$Pr = 0.7$	5/2	0.354	0.353
$n_\lambda = n_\mu = 1$	2	0.354	0.353
	3/2	0.354	0.353
	2/4	0.354	0.353
	1/2	0.354	0.353
	1/3	0.354	0.353

Table 4. $Nu_x/(Gr_x)^{1/4}$ for free convection of gas on a vertical isothermal flat plate

	T_w/T_∞	Ar $Pr = 0.622$ $n_\mu = 0.72$ $n_\lambda = 0.73$	H_2 $Pr = 0.68$ $n_\mu = 0.68$ $n_\lambda = 0.80$	Air $Pr = 0.7$ $n_\mu = 0.68$ $n_\lambda = 0.81$	N_2 $Pr = 0.71$ $n_\mu = 0.67$ $n_\lambda = 0.76$	CO $Pr = 0.72$ $n_\mu = 0.71$ $n_\lambda = 0.83$	O_2 $Pr = 0.733$ $n_\mu = 0.694$ $n_\lambda = 0.86$	Water vapour $Pr = 1$ $n_\mu = 1.04$ $n_\lambda = 1.185$
$Nu_{x,\infty}/(Gr_{x,\infty})^{1/4}$	3	0.306	0.336	0.342	0.333	0.348	0.359	0.452
	5/2	0.311	0.338	0.344	0.337	0.349	0.359	0.443
	2	0.319	0.341	0.347	0.341	0.352	0.359	0.432
	3/2	0.326	0.345	0.350	0.347	0.354	0.359	0.419
	≈1	0.339	0.350	0.354	0.356	0.358	0.360	0.401
	3/4	0.345	0.351	0.355	0.359	0.358	0.358	0.390
	1/2	0.354	0.352	0.357	0.363	0.359	0.354	0.376
	1/3	0.363	0.353	0.359	0.367	0.359	0.351	0.362
$Nu_{x,w}/(Gr_{x,w})^{1/4}$	3	0.353	0.351	0.354	0.362	0.358	0.354	0.377
	5/2	0.350	0.351	0.354	0.361	0.358	0.355	0.381
	2	0.347	0.351	0.354	0.360	0.358	0.356	0.386
	3/2	0.344	0.350	0.354	0.358	0.358	0.358	0.392
	≈1	0.339	0.350	0.354	0.356	0.358	0.360	0.401
	3/4	0.332	0.347	0.351	0.351	0.355	0.359	0.409
	1/2	0.323	0.342	0.347	0.344	0.352	0.358	0.421
	1/3	0.314	0.338	0.343	0.338	0.348	0.356	0.434

those reported in ref. [1], as shown in Table 3. $Nu_{x,\infty}/(Gr_{x,\infty})^{1/4}$ and $Nu_{x,w}/(Gr_{x,w})^{1/4}$ for various gases at different values of T_w/T_∞ have been calculated according to equations (33)–(36) respectively and are summarized in Table 4.

6. CONCLUSION

The following points may be concluded from the discussion.

(1) The method proposed for analysing the laminar free convection of monoatomic and diatomic gases, air and water vapour along vertical isothermal flat plates with considerations of variable fluid properties can yield reliable results.

(2) The analysis presented here extends the former ones reported in the literature by Sparrow and Gregg [1], Brown [2] and Gray and Giogini [3].

(3) The well-known relations, equations (34) and (36), hold true not only for the case $T_w/T_\infty \to 1$ such that the Boussinesq approximation applies, but also for the gases with $n_\mu = n_\lambda = 1$.

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EFFET DE LA VARIATION DES PROPRIETES THERMOPHYSIQUES SUR LA CONVECTION LAMINAIRE NATURELLE DES GAZ

Résumé—La conductivité thermique et la viscosité dynamique sont supposées varier en fonction de la température absolue suivant une loi puissance simple, la masse volumique est inversement proportionnelle à la température absolue pour une pression constante et le nombre de Prandtl est constant. Les équations de la convection laminaire naturelle sont transformées en équations différentielles adimensionnelles par des hypothèses de similitude. Elles sont résolues par une méthode de tir. Les résultats numériques sont donnés et discutés.

DER EINFLUSS VARIABLER THERMOPHYSIKALISCHER EIGENSCHAFTEN AUF DIE LAMINARE FREIE KONVEKTION EINES GASES

Zusammenfassung—Es wird angenommen, daß die Wärmeleitfähigkeit und die dynamische Viskosität sich entsprechend einem einfachen Potenzgesetz mit der absoluten Temperatur ändern, die Dichte hängt bei konstantem Druck umgekehrt proportional von der absoluten Temperatur ab, während die Prandtl-Zahl konstant ist. Die Grundgleichungen für die laminare freie Konvektion eines Gases werden mit Hilfe einer Ähnlichkeitstransformation in dimensionslose gewöhnliche Gleichungen überführt. Sie werden mit Hilfe eines "shooting"-Verfahrens gelöst. Die Ergebnisse der numerischen Berechnung werden zusammen mit einer eingehenden Diskussion mitgeteilt.

ВЛИЯНИЕ ПЕРЕМЕННЫХ ТЕПЛОФИЗИЧЕСКИХ СВОЙСТВ НА СВОБОДНУЮ КОНВЕКЦИЮ ПРИ ЛАМИНАРНОМ ТЕЧЕНИИ ГАЗА

Аннотация—Предполагается, что теплороводность и динамическая вязкость зависят от абсолютной температуры согласно простому степенному закону. Принято, что при постоянном давлении плотность обратно пропорциональна абсолютной температуре, а число Прандтля не изменяется. Определяющие уравнения для свободной конвекции нри ламинарном течении газа методом преобразования подобия сводятся к обыкновенным безразмерным дифференциальным уравнениям и решаются методом пристрелки. Наряду с результатами численного расчета приводится их детальный анализ.